IBM IEEE CAS/EDS
AI Compute Symposium 2022
October 12-13

Designing Highly Reliable Systems Using Random Devices

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Neurons are prime examples of efficiency, achieving outstanding *communication reliabilities*, although relying on *random ion channels*.

Aiming to *bridge from biology to circuits*, we will show here how:

- statistical results about consecutive systems ...
- combined with a Binet-like formula
- Fibonacci numbers of higher orders

lead to trivial reliability calculations for neuron-inspired optimal design schemes for *reliable communication*.

Structure

- Introduction
- Consecutive systems
- Optimal design
- Conclusions

ThUNDER² = Techniques for Unconventional Nano-Design in the Energy-Reliability Realm PN-III-P4-ID-PCE-2020-2495 nr. 238/07.04.2021

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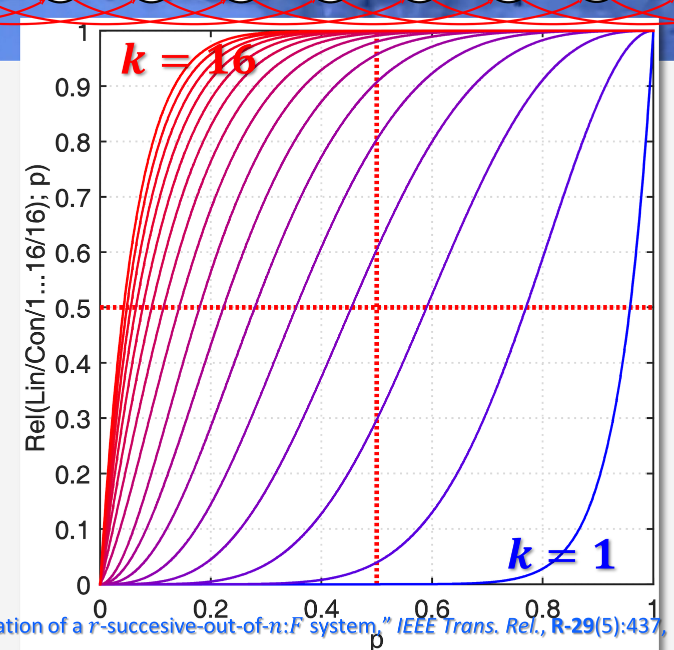

John M. Kontoleon, "Reliability determination of a r-succesive-out-of-n:F system," *IEEE Trans. Rel.*, **R-29**(5):437, Dec. 1980

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Abraham de Moivre (1738) – rephrased by James V. Uspensky (1937)

If $\beta_{m,k} = \sum_{j=0}^{\lfloor m/(k+1)\rfloor}(-1)^j C^j_{m-jk}(pq^k)^j$ (with $p = 1 - q$)

$P_{n,k} = 1 - \beta_{n,k} + q^k\,\beta_{n-k,k}$ (probability of a run of length k in n trials)

$$Rel(k,n;q) = \beta_{n,k} - q^k\,\beta_{n-k,k}$$

James C. Fu (1986) using the Markov chain method

$$M = \begin{pmatrix} p & q & 0 & \cdots & 0 & 0 \\ p & 0 & q & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ p & 0 & 0 & \cdots & 0 & q \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix}_{(k+1)\times(k+1)}$$

$$Rel(k,n;q) = (1,\dots,1,0)\times M^n \times (1,\dots,1,0)^t$$

James V. Uspensky, *Introduction to Mathematical Probability*, McGraw-Hill: New York, NY, USA, 1937
James C. Fu, "Reliability of consecutive-k-out-of-n:F systems with $(k-1)$-step Markov dependence," *IEEE Trans Rel.*, **R-35**(5):602–606, Dec. 1986

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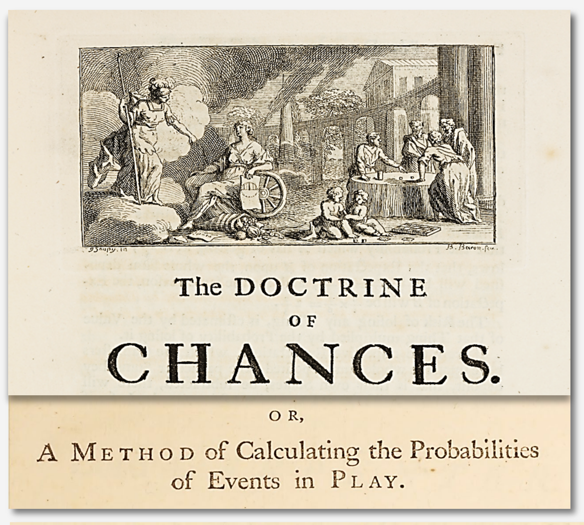
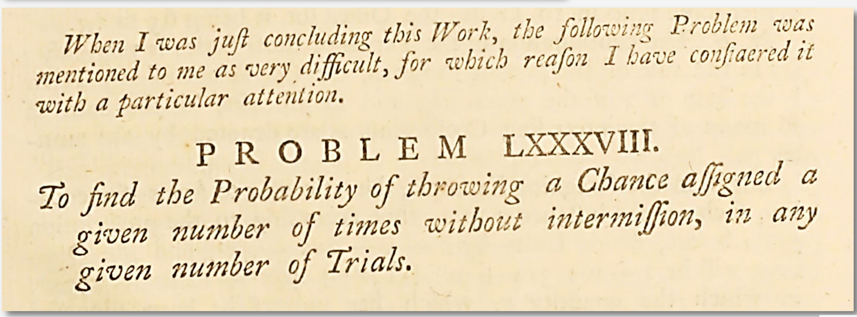

The DOCTRINE OF CHANCES. OR, A METHOD of Calculating the Probabilities of Events in PLAY.


When I was just concluding this Work, the following Problem was mentioned to me as very difficult, for which reason I have considered it with a particular attention.

PROBLEM LXXXVIII.

To find the Probability of throwing a Chance assigned a given number of times without intermission, in any given number of Trials.

Abraham de Moivre, *The Doctrine of Chances* (2nd ed.), H. Woodfall: London, UK, 1738
https://catalog.lindahall.org/discovery/delivery/01LINDAHALL_INST:LHL/1287366660005961

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$Rel(Lin/Con/k/n;0.5) = \frac{F^{(k)}_{n+2}}{2^n}$ Philippou 1985

$F^{(k)}_n = F^{(k)}_{n-1} + F^{(k)}_{n-2} + \cdots + F^{(k)}_{n-k}$ d'Ocagne 1884

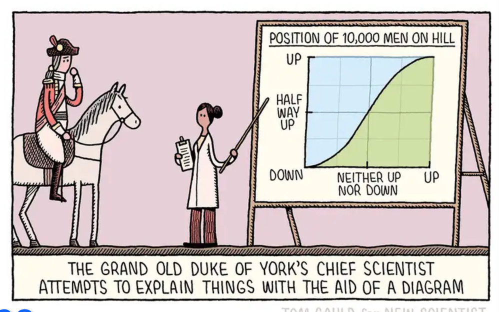
$x^k - x^{k-1} - x^{k-2} - \cdots - x^2 - x - 1 = 0$

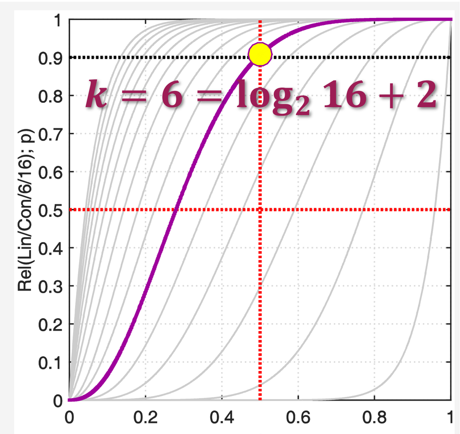
$\alpha_k = 2\left[1 - \sum_{i\ge1}\frac{1}{i}\binom{(k+1)i-2}{i-1}\frac{1}{2^{(k+1)i}}\right]$ Wolfram 1998

$2 - \frac{1}{2^{k-1}} < 2 - \frac{1}{2^{k-\frac{k}{2}-\frac{k^2}{2^k}}} < \alpha_k < 2 - \frac{1}{2^{k-\frac{k}{2}}} < 2 - \frac{1}{2^k}$

$F^{(k)}_n = \mathrm{rnd}\left[\frac{\alpha_k - 1}{2+(k+1)(\alpha_k-2)}\alpha_k^{n-1}\right]$ Du 2008, Dresden 2009

$\lim_{n\to\infty}\frac{F^{(\log_2 n+\delta)}_{n+2}}{2^n} = e^{-1/2^{1+\delta}}$ which gives $k_{min} = \log_2 n + 2|3$



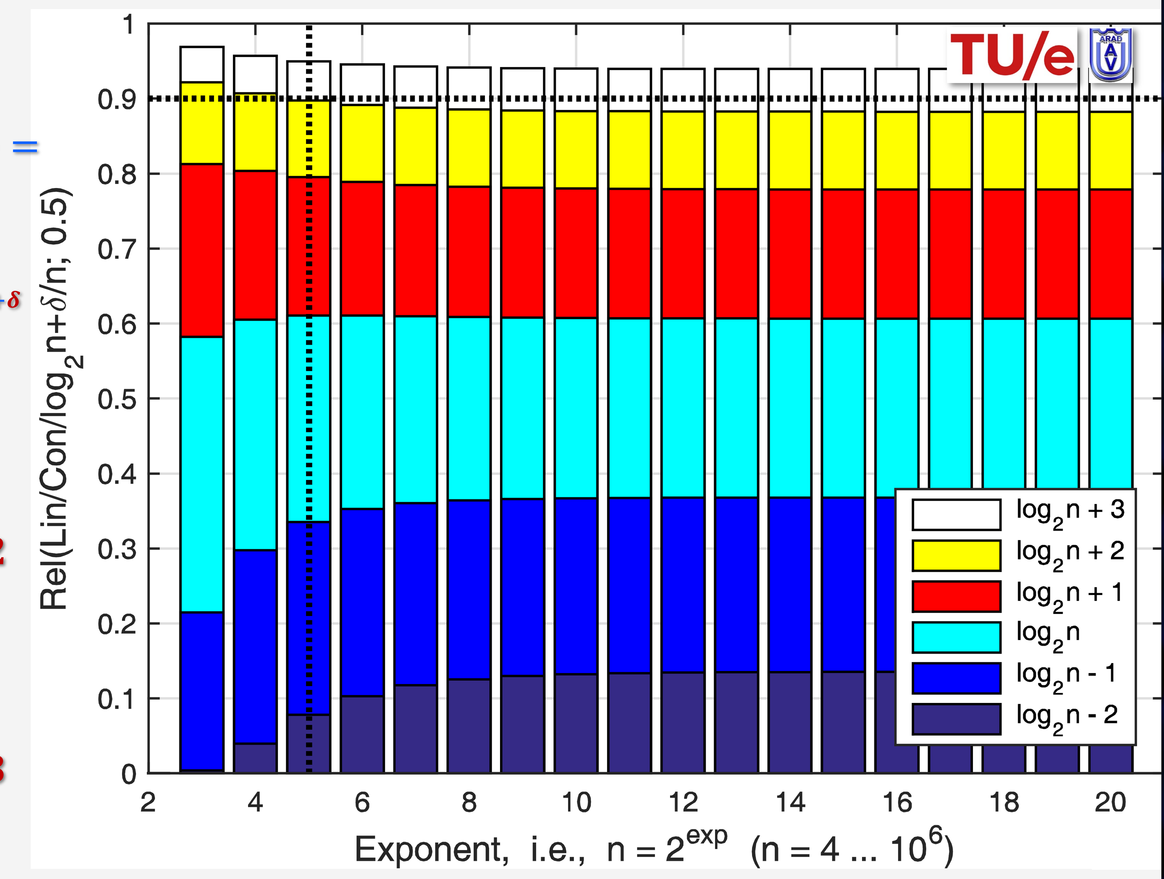


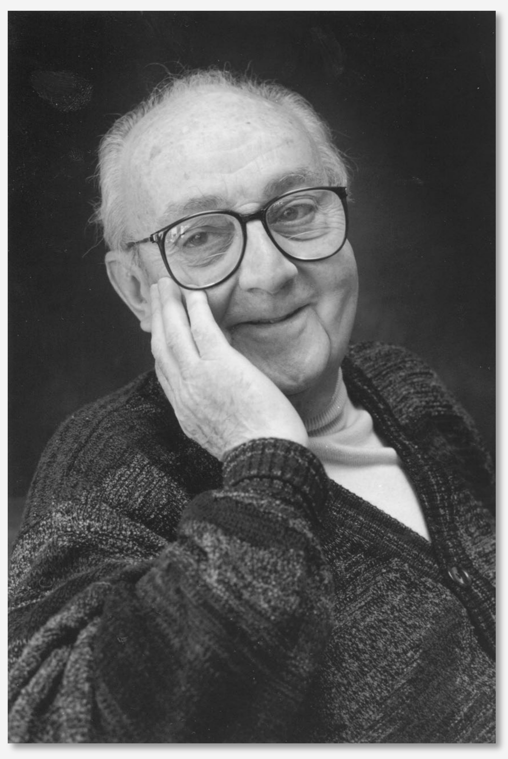
$Rel(Lin/Con/k/n;0.5) = \frac{F^{(k)}_{n+2}}{2^n}$

$\lim_{n\to\infty}\frac{F^{(\log_2 n+\delta)}_{n+2}}{2^n} = e^{-1/2^{1+\delta}}$

For $n \le 16$
$k_{min} = \log_2 n + 2$

For $n \ge 32$
$k_{min} = \log_2 n + 3$





- [...] all models are approximations
- ***Essentially, all models are wrong, but some are useful***
- [...] the approximate nature of the model must always be borne in mind

George E. P. Box, "Science and statistics," *J. Amer. Stat. Assoc.*, **71**(356): 791–799, Dec. 1976

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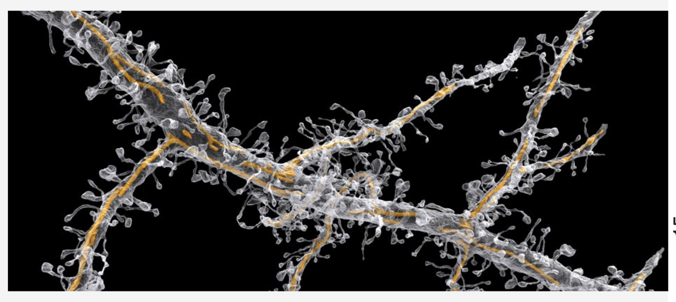
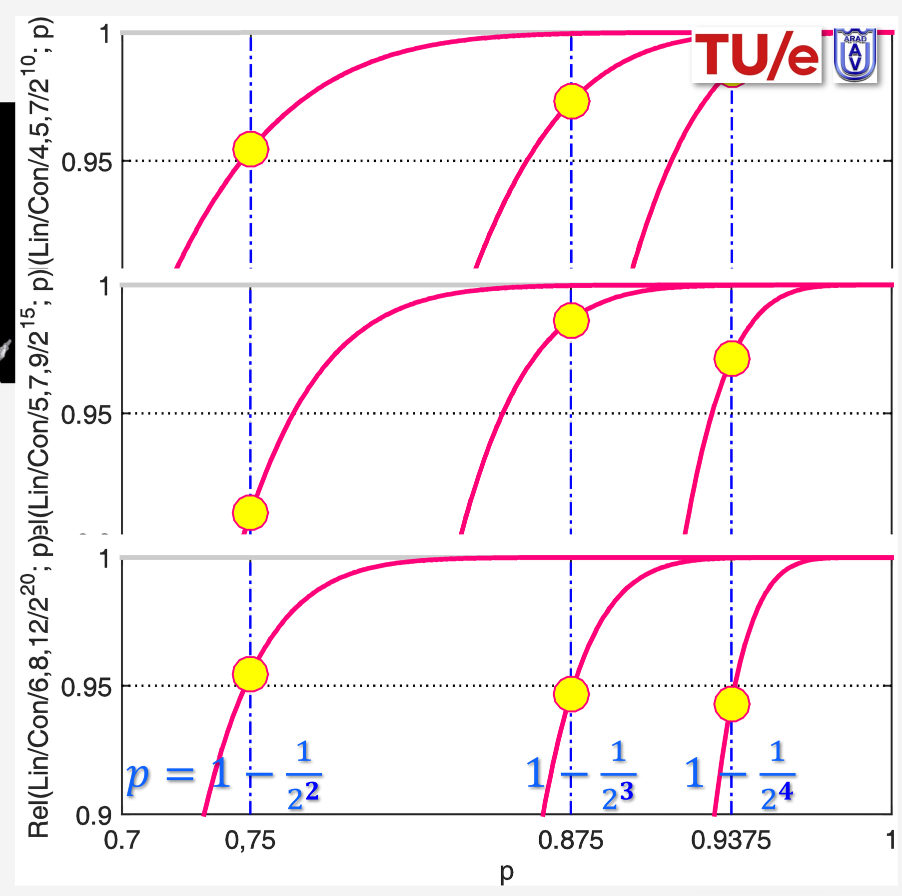


Axons could be modeled as 2D consecutive:

- (m,k)-out-of-(m,n)
- $n = 10^2 \dots 10^6$
- $m = 2, 3, 4$

$$k_{min} = \left\lceil \frac{\log_2 n + 3}{m} \right\rceil$$



Our method for optimizing linear consecutive systems:

- avoids computing the reliability polynomial
- relies on a (very) simple Binet-like formula
- $Rel > 90\%$ with $p = 1/2$ (!) for all $n > 4$
- is being extended to 2D (axons)
- technology evaluations are underway

THANK YOU

Research supported by a grant of the Romanian Ministry of Education and Research, CNCS-UEFISCDI, project no. PN-III-P4-ID-PCE-2020-2495, within PNCDI III (ThUNDER² = *Techniques for Unconventional Nano-Designing in the Energy-Reliability Realm*)