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Abstract

As is known, solitons are eigenmodes of problems arising in dispersive nonlinear media. Examples of such problems can be Bose-Einstein condensates, optical waveguides and a system of carbon nanotubes. Note that the latter system was considered in the approximation when carbon nanotubes are evenly distributed over the sample volume. In addition, an equation was obtained for carbon nanotube systems, which is a generalization of the nonlinear Schrodinger equation.

Nonlinear Schrodinger equations (NUS), with focusing or defocusing nonlinearity, are universal models that make it possible to consider solitons of all personal types. Varieties of solitons can be radically supplemented in spatially inhomogeneous conditions. Usually nonlinear lattices are used, i.e. periodically spatially modulated changes in the nonlinearity coefficient.

INTRODUCTION

An extension of the nonlinear Schrodinger equations was introduced by Laskin in , in the form of a fractional equation:

$$\varepsilon_s(p) = \pm \gamma_0 \sqrt{1 + 4 \cos(ap) \cos\left(\frac{\pi s}{m}\right) + 4 \cos^2\left(\frac{\pi s}{m}\right)}$$

where $s=1, 2 \dots m$ the nanotube has the type $(m,0)$ $\gamma_0 \approx 2.7$ eV,
 $a = 3b/2\hbar$ $b = 0.142$ the distance between neighboring carbon atoms.

In this case, the effective equation obtained in is:

$$i \frac{\partial B}{\partial z} + \frac{D}{2} \frac{\partial^2 B}{\partial \tau^2} - \frac{\omega_0^2}{c^2} B \sum_q \sum_l b_q \frac{(-1)^l q^{2l+1} |B|^{2l}}{l! (l+1)! 2^{2l}} = 0,$$

where here ω_0 - is the characteristic frequency, τ - is the time in the accompanying coordinate system, D - is the dispersion of group velocities, R - is the nonlinearity coefficient. Note that the electromagnetic pulse field has the form:

$$(z, t) = \text{Re}\{B(z, t) \cdot \exp[i(kz - \omega t)]\}$$

where A is the vector potential of the electromagnetic pulse: $A=(0, A(z,t), 0, 0)$.

In the resulting effective equation, the replacement was performed:

$$\frac{\partial^2 B}{\partial \tau^2} \rightarrow \frac{d^\alpha B}{d\tau^\alpha}$$

here α is the order of the fractional derivative $1 < \alpha < 2$.

$$\left(-\frac{\partial^2}{\partial x^2}\right)^{\alpha/2} E = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dp |p|^\alpha \int_{-\infty}^{+\infty} d\xi e^{ip(x-\xi)} E(\xi).$$

The main result obtained is shown in the figure below:

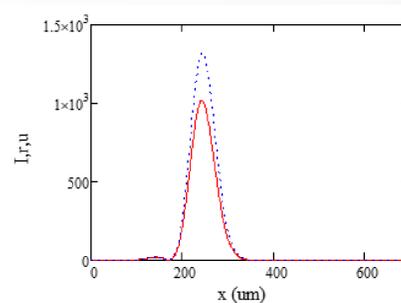


Fig. 1. the solid curve $\alpha=1.6$, the dashed curve $\alpha=1.7$. I – intensity is expressed in relative units.

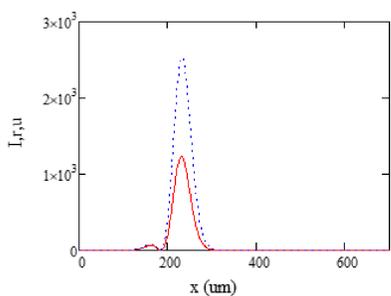


Fig. 2. the solid curve $\alpha=1.6$, the dashed curve $\alpha=1.9$. I – intensity is expressed in relative units.

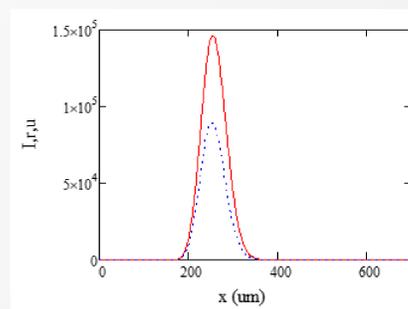


Fig. 3. the solid curve $\alpha=1.6$, the dashed curve $\alpha=1.5$. I – intensity is expressed in relative units.

Conclusions

Based on the presented graphs, it can be concluded that the order of the fractional derivative, i.e. the degree of disorder of the medium, significantly affects the optical impulse dynamics.