Dissipation of incident wave energy by two floating horizontal porous plates over a trench-type bottom

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Abstract

A mathematical model for wave interaction with a pair of plates over a trench-type bottom topography is developed. This problem is modelled based on Darcy's law for flow past a porous structure. Havelock's expansions of water wave potentials, suitable matching conditions, and the algebraic least-square method handle the boundary value problem. The role of the horizontal porous plates is studied by analysing the scattering coefficients, energy loss, hydrodynamic wave force and free surface elevation through graphs which show a periodic oscillatory pattern as a function of gap length. The study reveals that if the porous effect parameter of plates or the length of plates is increased, more energy is dissipated, and the plates experience less wave force. The free surface elevation on the lee side is decreased. The findings from the present model are likely to help understand the role of breakwaters in engineering applications.

Mathematical formulation

PDE

$$\frac{\partial^2 \phi_j}{\partial^2 x} + \frac{\partial^2 \phi_j}{\partial^2 y} - l^2 \phi_j = 0, \quad \text{in each region } R_j,\ j = 1, ..., 5 \text{ and } l = k_0 \cos\theta_1, \tag{1}$$

Boundary conditions:

Free surface boundary condition
$$\frac{\partial \phi_j}{\partial y} + K\phi_j = 0 \quad \text{on } y = 0, \quad j = 1, 3, 5 \text{ where } K = \omega^2/g, \tag{2}$$

No-flow condition
$$\frac{\partial \phi_j}{\partial y} = 0 \quad \text{at } y = h_1 \quad \text{and } j = 1, 2, 4, 5, \qquad \frac{\partial \phi_3}{\partial y} = 0 \quad \text{at } y = h_2, \tag{3}$$

Conditions on the porous plates
$$\frac{\partial \phi_2}{\partial y} = ik_0 G_1 \phi_2 \quad \text{on } y = 0, \qquad \frac{\partial \phi_4}{\partial y} = ik_0 G_2 \phi_4 \quad \text{on } y = 0, \tag{4}$$

Matching conditions
$$\phi_1(-b, y) = \phi_2(-b, y), \quad \phi_{1x}(-b, y) = \phi_{2x}(-b, y) \quad \text{for } 0 < y < h_1, \tag{5}$$

$$\phi_2(-a, y) = \phi_3(-a, y), \quad \phi_{2x}(-a, y) = \phi_{3x}(-a, y) \quad \text{for } 0 < y < h_1, \tag{6}$$

$$\phi_3(a, y) = \phi_4(a, y), \quad \phi_{3x}(a, y) = \phi_{4x}(a, y) \quad \text{for } 0 < y < h_1, \tag{7}$$

$$\phi_4(c, y) = \phi_5(c, y), \quad \phi_{4x}(c, y) = \phi_{5x}(c, y) \quad \text{for } 0 < y < h_1, \tag{8}$$

$$\phi_{3x}(-a, y) = 0, \quad \phi_{3x}(+a, y) = 0 \quad \text{for } h_1 < y < h_2. \tag{9}$$

Schematic Diagram

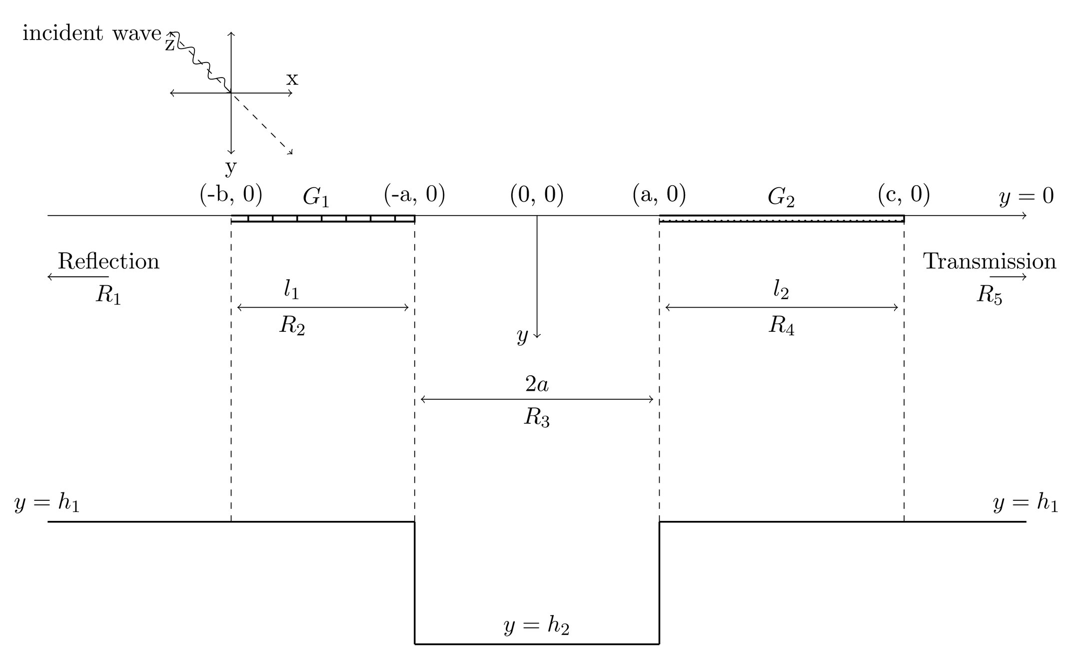

Fig. 1: Schematic of physical problem

Method of solution

- Eigenfunction expansion method
- System of equation $[(8m_1 + 2m_2) \times (8N + 8)]$
- Algebraic least-square method

Numerical Results

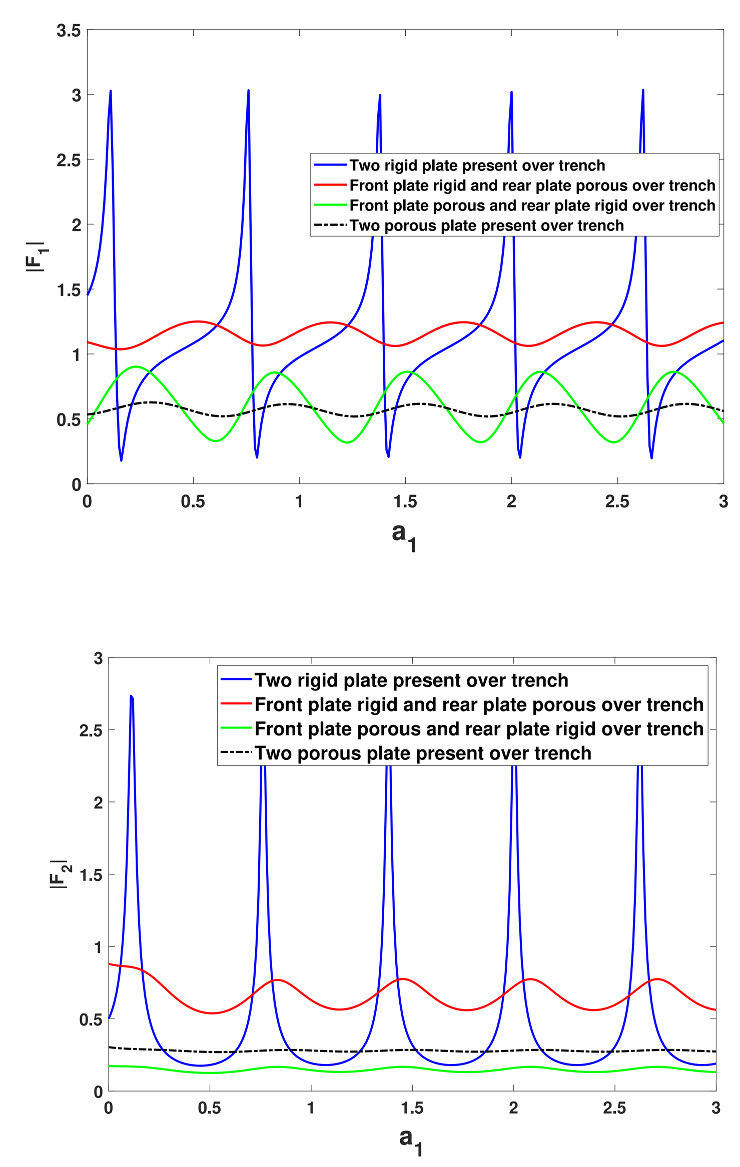

Fig. 2: Variations in (a) $|F_1|$ and (b) F_2 versus a_1 for different plate cases with fixed values of $Kh_1 = 2.5, \theta_1 = 0^0, H_2 = 2$

Numerical Results

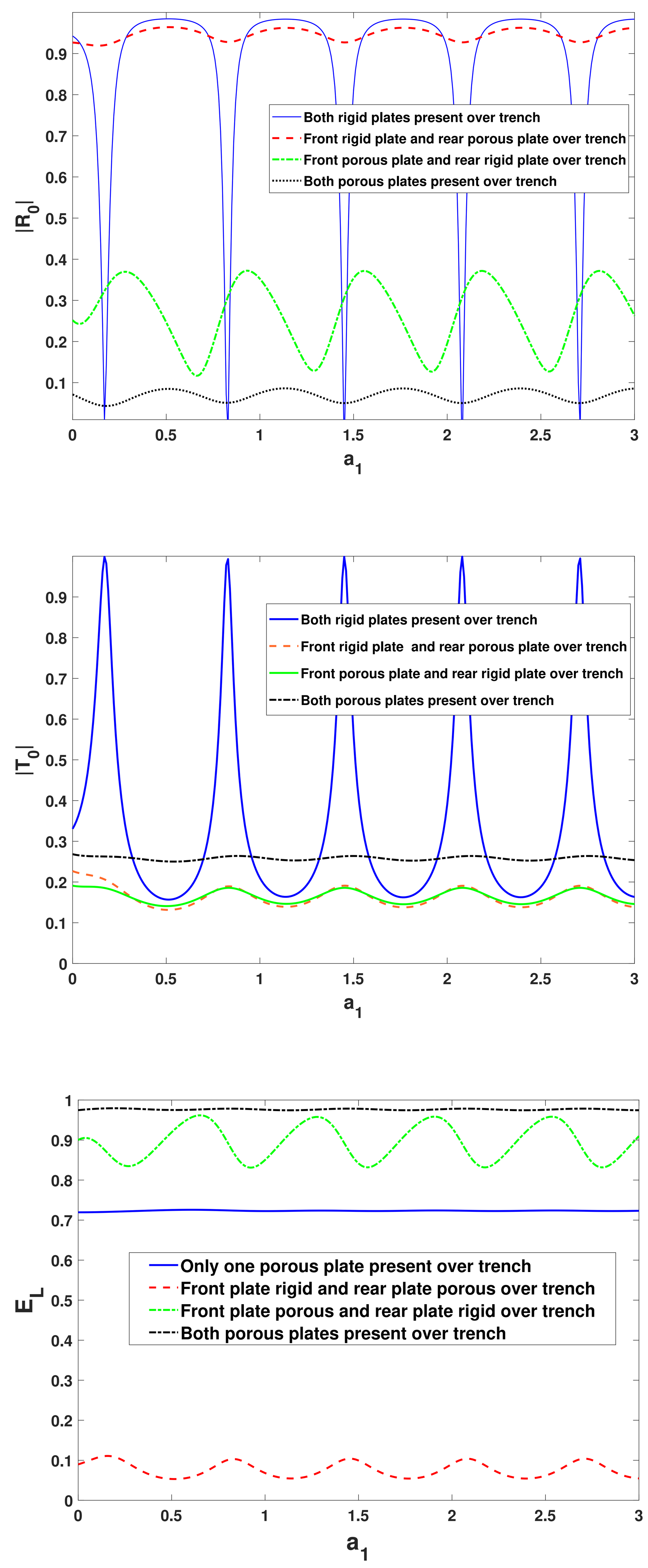

Fig. 3: Variations in (a) $|R_0|$, (b) $|T_0|$ and (c) E_L versus a_1 for different types of plates with fixed values of $Kh_1 = 2.5, \theta_1 = 0, H_2 = 2$

Conclusions

- The scattering of normal incident waves by two horizontal porous plates over a trench-type sea bed is investigated numerically using the algebraic least-square method.
- The role of the horizontal porous plates is studied through graphs by analysing the scattering coefficients, dissipated energy and wave force on plates.
- It is studied that increasing one plate's porosity/length is less effective than increasing both plates' porosity/length.
- In this study, it is noted that the wave load on the porous plates is much less than on rigid plates.
- The length of the floating horizontal porous plates, porous effect parameters and gap between the plates play an important role in reducing the free surface elevation in the lee side region due to the dissipation of wave energy by the plates.

Acknowledgement

Sunita Choudhary is thankful to the University Grants Commission (UGC), Govt. of India, for fellowship (Ref. No. JUNE18-419784) for pursuing Ph.D. degree at the Indian Institute of Technology Ropar, India.

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