Fractional Mathematical Modelling of Malaria Disease with Treatment & Insecticides

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Abstract

The dynamics of malaria illness among humans and vectors are examined in this study. The problem is described using nonlinear ODEs that are then generalized using the Atangana-Baleanu fractional derivative. Optimal control strategies have been done. From the graphical results, it can be noticed that the control parameters drastically decrease the number of infected human and vector population which will off course minimize the spread of infection among the human population. Moreover, the use of bednets and insecticides can reduce the spread of infection dramatically while the impact of medication and treatment on the control of infection is comparatively less.

Compartmental Model Diagram of Malaria Disease

The Model Diagram of Malaria Disease from which the mathematical model has been developed,

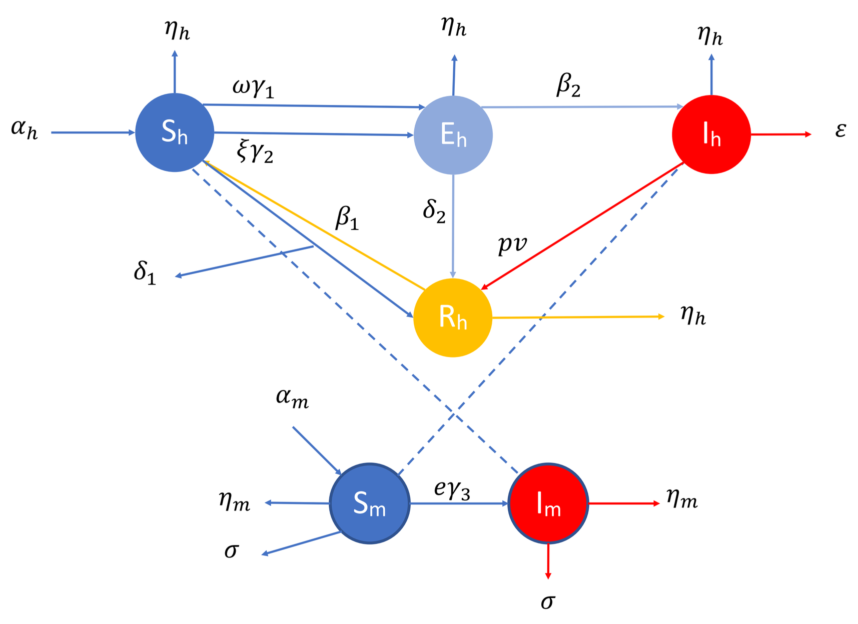
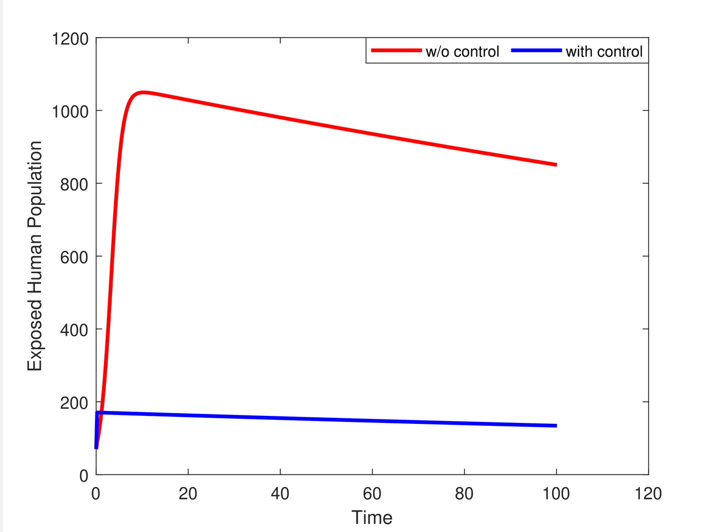


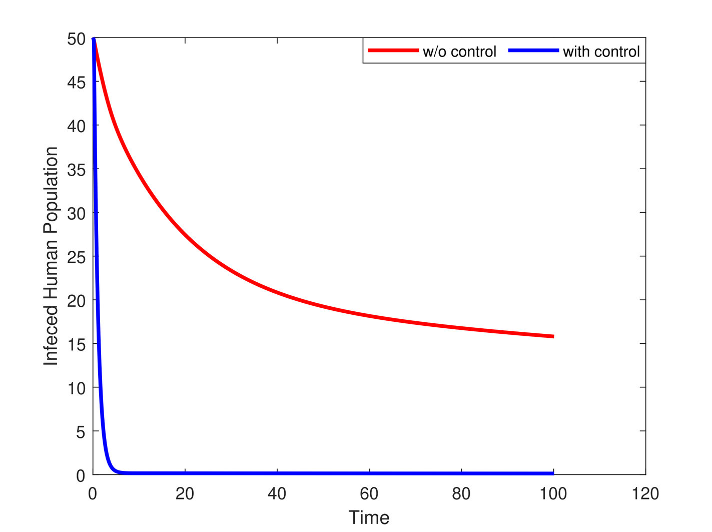
Figure 1. Compartmental Diagram for the transmission dynamics of the mathematical model of Malaria.

Mathematical model of Malaria Disease

The dynamical system below is developed from the diagram 1

$$
\begin{aligned}
{}^{ABC}_{0}\mathcal{D}^{\gamma}_{t}\mathcal{S}_h(t) &= \alpha_h + \beta_1\mathcal{R}_h - \omega\gamma_1\mathcal{S}_h\mathcal{I}_h - \xi\gamma_2\mathcal{S}_h\mathcal{I}_m - (\delta_1+\eta_h)\mathcal{S}_h,\\
{}^{ABC}_{0}\mathcal{D}^{\gamma}_{t}\mathcal{E}_h(t) &= \omega\gamma_1\mathcal{S}_h\mathcal{I}_h + \xi\gamma_2\mathcal{S}_h\mathcal{I}_m - (\eta_h+\beta_2+\delta_2)\mathcal{E}_h,\\
{}^{ABC}_{0}\mathcal{D}^{\gamma}_{t}\mathcal{I}_h(t) &= \beta_2\mathcal{E}_h - (\eta_h+\varepsilon+pv)\mathcal{I}_h,\\
{}^{ABC}_{0}\mathcal{D}^{\gamma}_{t}\mathcal{R}_h(t) &= pv\mathcal{I}_h + \delta_1\mathcal{S}_h + \delta_2\mathcal{E}_h - (\beta_1+\eta_h)\mathcal{R}_h,\\
{}^{ABC}_{0}\mathcal{D}^{\gamma}_{t}\mathcal{S}_m(t) &= \alpha_m - e\gamma_3\mathcal{S}_m\mathcal{I}_h - (\eta_m+\sigma)\mathcal{S}_m,\\
{}^{ABC}_{0}\mathcal{D}^{\gamma}_{t}\mathcal{I}_m(t) &= e\gamma_3\mathcal{S}_m\mathcal{I}_h - (\eta_m+\sigma)\mathcal{I}_m.
\end{aligned}
\tag{1}
$$





Adjoint system and control variables based on Optimal Control Strategies

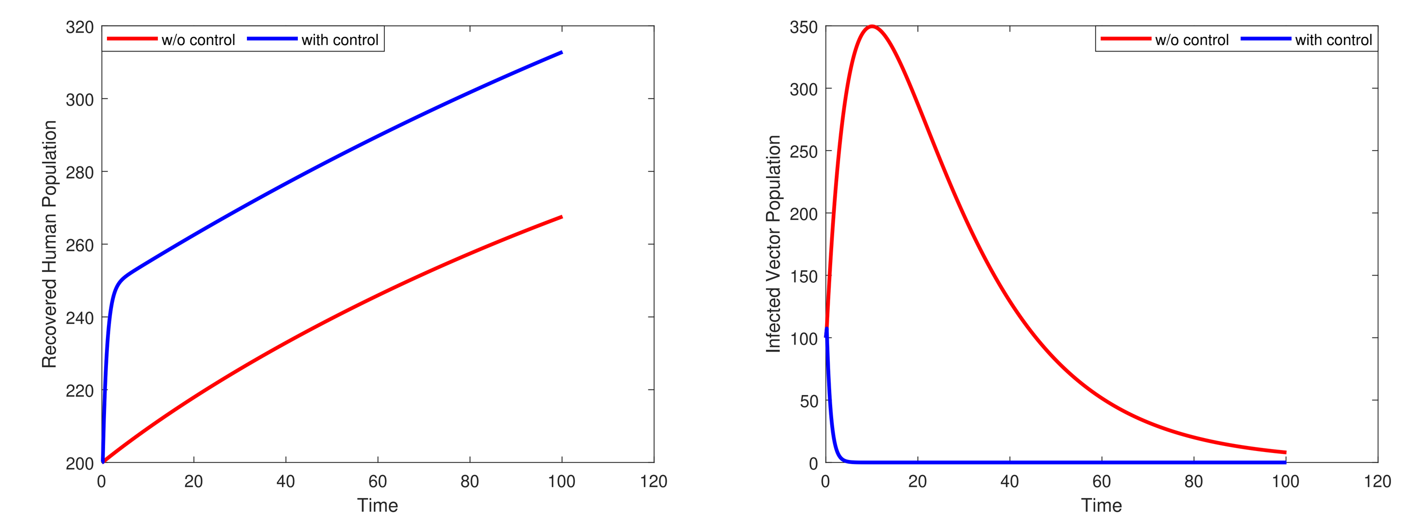
Adjoint system is given by,

$$
\begin{aligned}
{}^{ABC}_{0}\mathcal{D}^{\gamma}_{t}\lambda_{\mathcal{S}_h} &= \delta_1\lambda_{\mathcal{R}_h} - \lambda_{\mathcal{S}_h}\big(\delta_1+\eta_h-\mathcal{I}^*_m\gamma_2(\mathbf{u}_2-1)-\mathcal{E}^*_h\gamma_1\omega\xi(\mathbf{u}_1-1)\big)\\
&\quad - \lambda_{\mathcal{E}_h}\big(\mathcal{I}^*_m\gamma_2(\mathbf{u}_2-1)\\
&\quad + \mathcal{E}^*_h\gamma_1\omega\xi(\mathbf{u}_1-1)\big),\\
{}^{ABC}_{0}\mathcal{D}^{\gamma}_{t}\lambda_{\mathcal{E}_h} &= \mathcal{X} + \beta_2\lambda_{\mathcal{I}_h} - \lambda_{\mathcal{E}_h}\big(\beta_2+\delta_2+\eta_h+\mathcal{S}^*_h\gamma_1\omega\xi(\mathbf{u}_1-1)\big)\\
&\quad + \delta_2\lambda_{\mathcal{R}_h} + \mathcal{S}^*_h\gamma_1\lambda_{\mathcal{S}_h}\omega\xi(\mathbf{u}_1-1),\\
{}^{ABC}_{0}\mathcal{D}^{\gamma}_{t}\lambda_{\mathcal{I}_h} &= \mathcal{Y} - \lambda_{\mathcal{I}_h}(\varepsilon+\eta_h+a_1\mathbf{u}_3+pv) + \lambda_{\mathcal{R}_h}(a_1\mathbf{u}_3+pv) - \mathcal{S}^*_m e\gamma_3\lambda_{\mathcal{I}_m}(\mathbf{u}_2-1)\\
&\quad + \mathcal{S}^*_m e\gamma_3\lambda_{\mathcal{S}_m}(\mathbf{u}_2-1),\\
{}^{ABC}_{0}\mathcal{D}^{\gamma}_{t}\lambda_{\mathcal{R}_h} &= \beta_1\lambda_{\mathcal{S}_h} - \lambda_{\mathcal{R}_h}(\beta_1+\eta_h),\\
{}^{ABC}_{0}\mathcal{D}^{\gamma}_{t}\lambda_{\mathcal{S}_m} &= -\lambda_{\mathcal{S}_m}\big(\eta_m+\sigma+a_2\mathbf{u}_4-\mathcal{I}^*_h e\gamma_3(\mathbf{u}_2-1)\big) - \mathcal{I}^*_h e\gamma_3\lambda_{\mathcal{I}_m}(\mathbf{u}_2-1),\\
{}^{ABC}_{0}\mathcal{D}^{\gamma}_{t}\lambda_{\mathcal{I}_m} &= Z - \lambda_{\mathcal{I}_m}(\eta_m+\sigma+a_2\mathbf{u}_4) - \mathcal{S}^*_h\gamma_2\lambda_{\mathcal{E}_h}(\mathbf{u}_2-1) + \mathcal{S}^*_h\gamma_2\lambda_{\mathcal{S}_h}(\mathbf{u}_2-1).
\end{aligned}
\tag{2}
$$

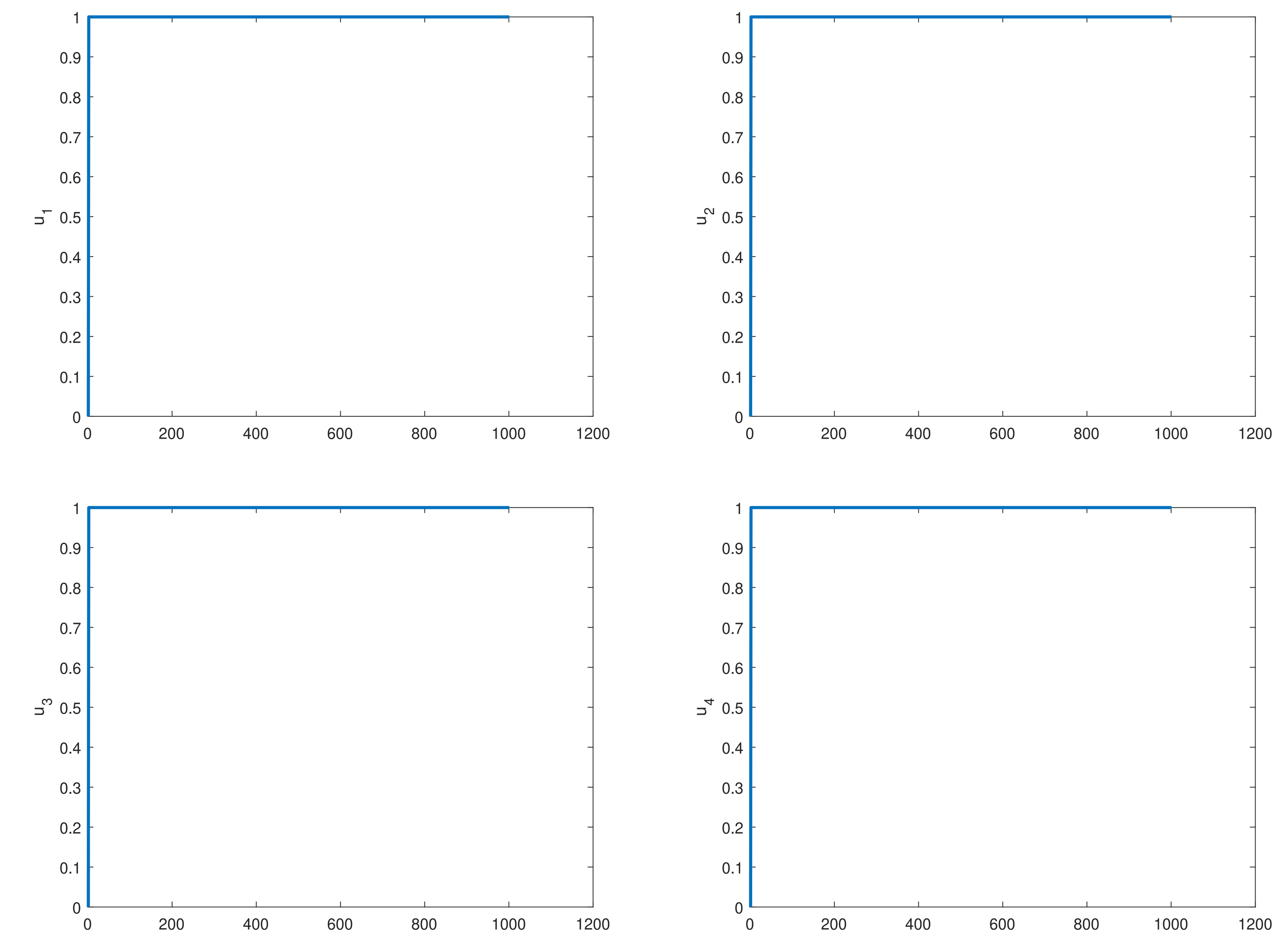
With control variables,

$$
\begin{aligned}
\mathbf{u}^*_1(t) &= max\left(min\left(\frac{\mathcal{E}^*_h\mathcal{S}^*_h\gamma_1\omega\xi\left(\lambda_{\mathcal{E}_h}-\lambda_{\mathcal{S}_h}\right)}{c_1},0\right),1\right),\\
\mathbf{u}^*_2(t) &= max\left(min\left(\frac{\mathcal{I}^*_m\mathcal{S}^*_h\gamma_2\lambda_{\mathcal{E}_h}-\mathcal{I}^*_m\mathcal{S}^*_h\gamma_2\lambda_{\mathcal{S}_h}+\mathcal{I}^*_h\mathcal{S}^*_m e\gamma_3\lambda_{\mathcal{I}_m}-\mathcal{I}^*_h\mathcal{S}^*_m e\gamma_3\lambda_{\mathcal{S}_m}}{c_2},0\right),1\right),\\
\mathbf{u}^*_3(t) &= max\left(min\left(\frac{\mathcal{I}^*_h,a_1\left(\lambda_{\mathcal{I}_h}-\lambda_{\mathcal{R}_h}\right)}{c_3},0\right),1\right),\\
\mathbf{u}^*_4(t) &= max\left(min\left(\frac{a_2(\mathcal{I}^*_m\lambda_{\mathcal{I}_m}+\mathcal{S}^*_m\lambda_{\mathcal{S}_m})}{c_4},0\right),1\right).
\end{aligned}
\tag{3}
$$

Results



As a result, all the figures give a clear picture of the implementation of the best control variables and their outcome from the model of malaria disease. The infection in the human as well as the vector population is reduced, and the recovery is also increased. Thus, the control variables used in this study shows good results, as a result these variables are recommend for doctors, researchers and scientists use for experimental work.



Conclusion

The dynamics of Malaria illness in a mutant human population are examined in this research. The mosquito population, which is regarded the major source for this form of sickness, is connected to its spread. The problem is formulated in classical order nonlinear coupled ODEs which are further generalized with ABC differential operator. Existence and uniqueness for the solution of the model are proved. Moreover, the model equilibria and their stability at each equilibrium point are calculated. Additionally, the optimal control of the present model is considered by introducing different control parameters which turns out to be more efficient in the reduction of infection in both infected human and vector populations. A numerical scheme is proposed for the solutions of the considered nonlinear problem. The graphical solutions are done through MATLAB software. To see the variation clearly, the simulation is carried out for both classical as well as fractional order. Moreover, global asymptotical stability has been shown through different graphs by slightly varying initial conditions.

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