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A Mathematical model and Design software of GM-Type Pulse tube Refrigerator

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Abstract

The R&D programme at NITR is directed towards the creation of a general-purpose analysis and design software with a modern user interface that can help scientists and engineers to build a pulse tube cryocooler of any cooling capacity at any given temperature. We also purpose to direct it especially to GM-type cryocooler with a hope that this will be further extended to other type of cryocoolers and other cryogenics equipment. The basic design approach in GM-type PTR is based on the trapezoidal profile of pressure vs time curve, the well-known isothermal model inside the pulse tube, heat transfer in the regenerator considering both primary (convection) and secondary (axial conduction) effects. The thermal and pneumatic descriptions are based on exergy analysis. Most relevant loss mechanisms such as regenerator convective losses (thermal and pneumatic), axial conduction, mixing in pulse tube have been taken into account. The software is written in MATLAB, has a decent graphical user interface and assists the designer not only in feeding input information but also in generating reports .

Universally available Softwares

Year	Software	Author	Institute/Organization	Reference
2015	StirlinGUIDE	Atray etal.	IITBombay, India	[1]
2013	Sharif PTR	Jahanbakhshi et al.	Sharif University of Technology, Iran	[2]
2010	SAGE	Gedeon D.	Gedeon Associates, Athens, OH, USA	[3]
2005	FZKPTR	Hofmann	Institute fur Technische Physik,D-76021 Karlsruhe , Germany	[4]
2002	MS*2	Mitchell and Bauwens	Berkeley, California, USA University of Calgary, Calgary, Alberta, Canada	[5]
1996	ARCOPTR	Roach and Kashani	NASA Ames Research Centre, Atlas Scientific ,Sunnyvale, USA	[6]
——	REGEN3.3,ISOHX,INERTANCE	Radebaugh etal.	NIST, Boulder, CO, USA	[7-9]

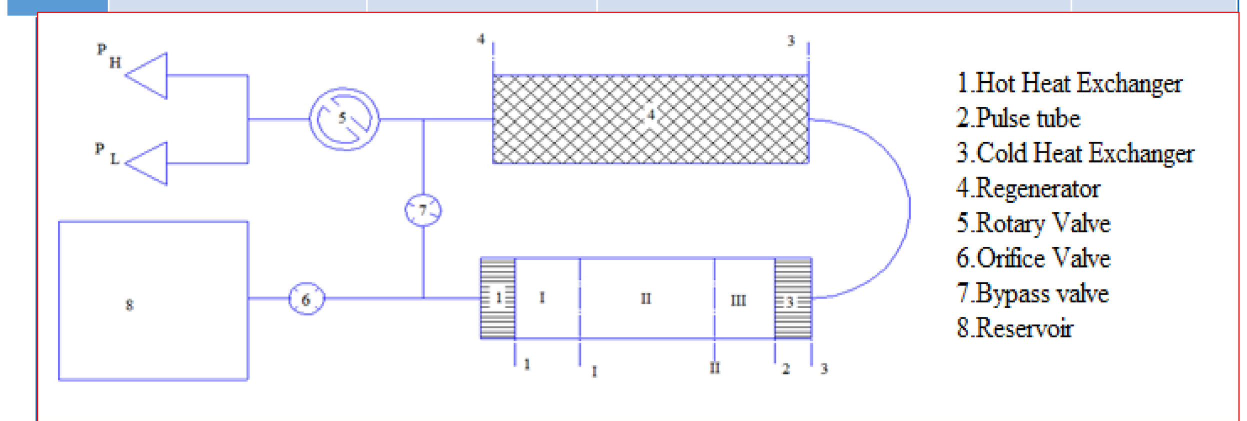


Fig1.Schematic diagram of GM-Type Pulse tube Refrigerator

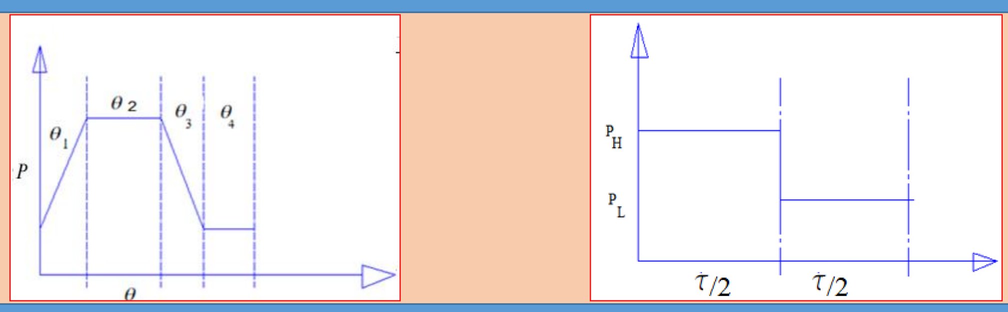


Fig-2.Pressure variation in Pulse tube and Rotary valve

Pressure variation in Rotary valve

[10]

$$P_5 = \begin{cases} P_H \left[0 < t \le \tau/2\right] \\ P_L \left[\tau/2 < t \le \tau\right] \end{cases} \qquad -(1)$$

Pressure variation in Pulse tube

$$P(t) = \begin{cases} P_L + \left(\frac{(P_H - P_L)}{\theta_1}\right) \times t\left[0 < t \le \theta_1\right] \\ P_H\left[\theta_1 < t \le \theta_2\right] \\ P_H - \left(\frac{(P_H - P_L)}{\theta_3 - \theta_2}\right) \times (t - \theta_2)\left[\theta_2 < t \le \theta_3\right] \\ P_L\left[\theta_3 < t \le \theta_4\right] \end{cases} \qquad -(2)$$

Mass flow rate in Orifice valve

$$\dot{m}_6 = \begin{cases} C_{d6}A_6\sqrt{\frac{2\times\gamma}{(\gamma-1)}\frac{P^2}{RT_H}\left[\left(\frac{P_8}{P}\right)^{2/\gamma} - \left(\frac{P_8}{P}\right)^{\frac{\gamma+1}{\gamma}}\right]} & P > P_8 \\ -C_{d6}A_6\sqrt{\frac{2\times\gamma}{(\gamma-1)}\frac{P_8^2}{RT_H}\left[\left(\frac{P}{P_8}\right)^{2/\gamma} - \left(\frac{P}{P_8}\right)^{\frac{\gamma+1}{\gamma}}\right]} & P < P_8 \end{cases} \qquad -(3)$$

Mass flow rate in Bypass valve

$$\dot{m}_7 = \begin{cases} C_{d7}A_7\sqrt{\frac{2\times\gamma}{(\gamma-1)}\frac{P^2}{RT_H}\left[\left(\frac{P_L}{P}\right)^{2/\gamma} - \left(\frac{P_L}{P}\right)^{\frac{\gamma+1}{\gamma}}\right]} & P > P_L \\ -C_{d7}A_7\sqrt{\frac{2\times\gamma}{(\gamma-1)}\frac{P_H^2}{RT_H}\left[\left(\frac{P}{P_H}\right)^{2/\gamma} - \left(\frac{P}{P_H}\right)^{\frac{\gamma+1}{\gamma}}\right]} & P < P_H \end{cases} \qquad -(4)$$

Ideal Refrigeration Power

$$\frac{dP_8}{dt} = \frac{\dot{m}_6 R T_H}{V_8} \qquad -(5)$$

$$\dot{m}_1 = \dot{m}_6 + \dot{m}_7 + \frac{V_1}{RT_H}\frac{dP}{dt} \qquad -(6)$$

$$m_I = m_{IO} + \int_0^t (-\dot{m}_1)dt \qquad -(7)$$

$$V_I = \frac{m_I R T_H}{P} \qquad -(8)$$

$$V_{II} = C_{IIO}P^{\left(-1/\gamma\right)} \qquad -(9)$$

$$V_{III} = V_2 - V_I - V_{II} \qquad -(10)$$

$$Q_{IDEAL} = f\oint PV_{III} \qquad -(11)$$

Regenerator ineffectiveness Loss

$$Q_{IL} = \left(\frac{\dot{m}_4 C_{vg}\left(T_H - T_C\right)}{NSTEP}\right)\frac{2}{2 + NTU} \qquad -(12)$$

Temperature swing Loss

$$DELTX = \left(\frac{\dot{m}_4}{NSTEP}\right)\left(T_H - T_C\right)\frac{C_{vg}}{\omega M_{matrix}\left(C_p\right)_{matrix}} \qquad -(13)$$

$$Q_{TS} = \left(\frac{\dot{m}_4}{NSTEP}\right)C_{vg}\frac{DELTX}{2} \qquad -(14)$$

Conduction Loss

$$(QC)_{PT} = K_{PT}A_{PT}\left(\frac{T_H - T_C}{L_{PT}}\right) \qquad -(15)$$

$$(QC)_R = K_R A_R\left(\frac{T_H - T_C}{L_R}\right) \qquad -(16)$$

$$(QC)_{matrix} = K_M A_M\left(\frac{T_H - T_C}{L_R}\right) \qquad -(17)$$

$$K_M = K_G\frac{K_x - (1-\alpha)}{K_x + (1-\alpha)} \qquad -(18)$$

$$K_X = \frac{1 + \left(K_R/K_G\right)}{1 - \left(K_R/K_G\right)} \qquad -(19)$$

$$(QC)_{TOTAL} = (QC)_{PT} + (QC)_R + (QC)_{matrix} \qquad -(20)$$

Void volume Loss at Cold end

$$Q_{VV} = (P_H - P_L)V_d \qquad -(21)$$

Regenerator Pressure drop Loss

$$Q_{PD} = \Delta P\left(\frac{PR + 1}{PR}\right)V_4 \qquad -(22)$$

$$\Delta P = \frac{f_{mean}}{2}\frac{L^2}{D_h}\frac{U^2{}_{max}}{\rho} \qquad -(23) \; [12]$$

[11]

$$f_{mean} = \frac{860(1-\alpha)}{\alpha} + \frac{2.2\alpha}{Re^{0.1}} \qquad -(24.a)$$

$$f_{mean} = \frac{1-\alpha}{\alpha}\left[\frac{570\times(1-\alpha)^2}{\alpha\times Re} + 3.5\right] \qquad -(24.b)$$

Radiation Loss

$$Q_{RD} = \frac{\sigma F A_C\left(T_H^4 - T_C^4\right)}{\frac{1}{e_c} + \frac{1}{e_s} - 1 + (N_S - 1)\left(\frac{2}{e_s} - 1\right) + \frac{1}{e_H} + \frac{1}{e_s} + 1} \qquad -(25)$$

Net Refrigeration Power

$$Q_{NET} = Q_{IDEAL} - \left(Q_{IL} + Q_{TS} + (QC)_{TOTAL} + Q_{VV} + Q_{PD} + Q_{RD}\right) \qquad -(26)$$

Work done by Compressor

[13]

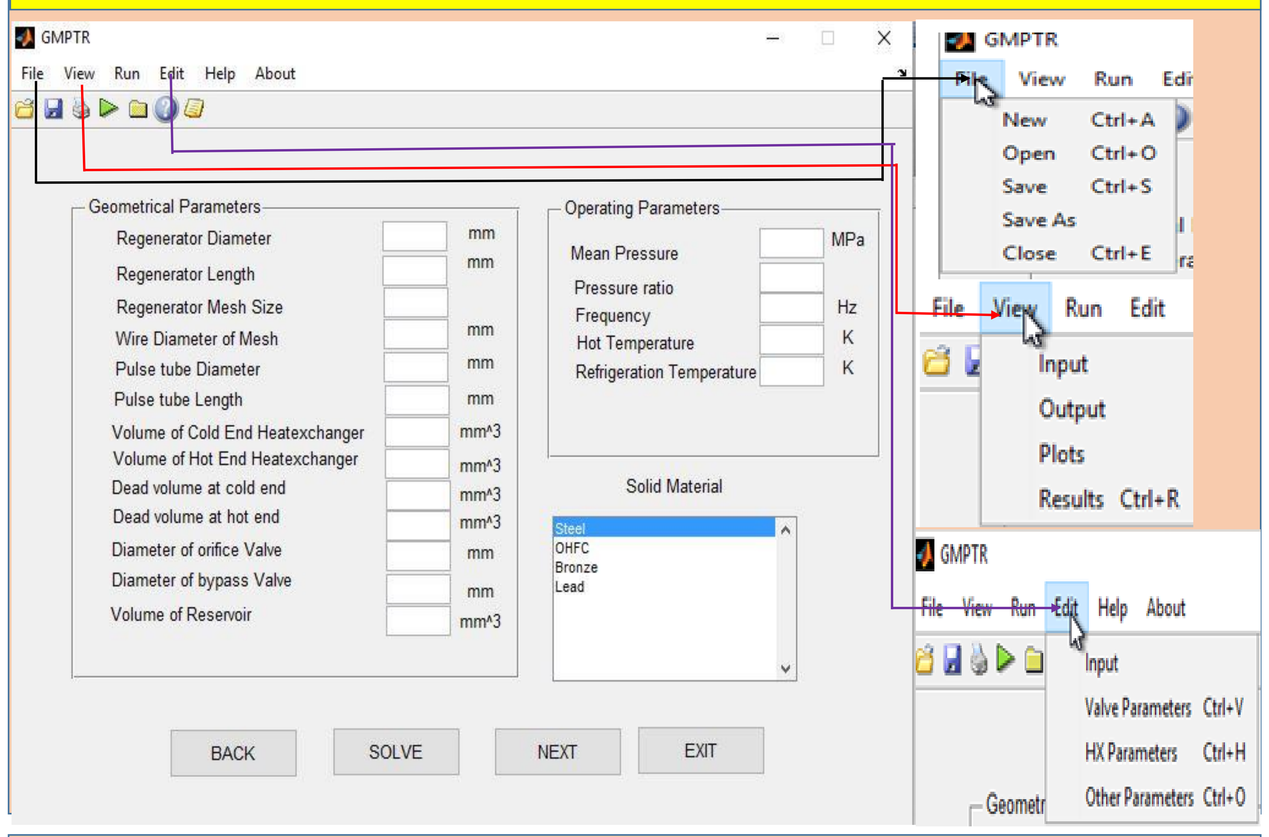
$$W = \dot{m}'_{comp}C_p T_h\left\{\left(\frac{P_H}{P_L}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\} \qquad -(27)$$

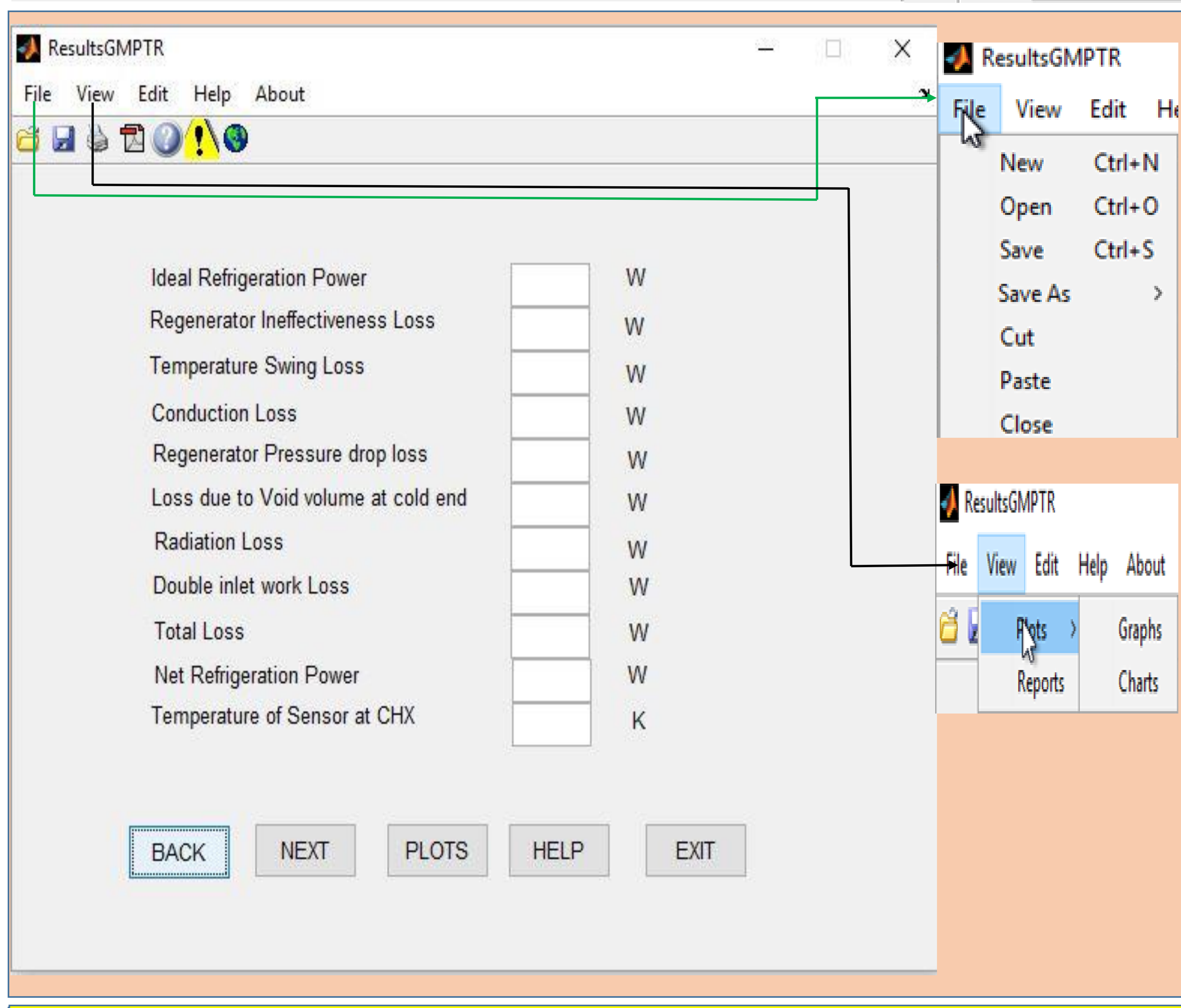
Exergy analysis Of each Control volume

$$(E_D)_{net} = f\left\{\oint \dot{m}_i\left[C_p(T_i - T_o) - T_o\left(C_p\ln\frac{T_i}{T_o} - R\ln\frac{P_i}{P_o}\right)\right] - \oint \dot{m}_e\left[C_p(T_i - T_o) - T_o\left(C_p\ln\frac{T_i}{T_o} - R\ln\frac{P_i}{P_o}\right)\right]dt + \sum\oint\left(1 - \frac{T_0}{T_k}\right)\dot{Q}_k dt - \oint W dt\right\} \qquad -(28)$$

[14]

Software overview





Conclusion

A software has been designed and developed to simulate the performance of Pulse tube Refrigerators.

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